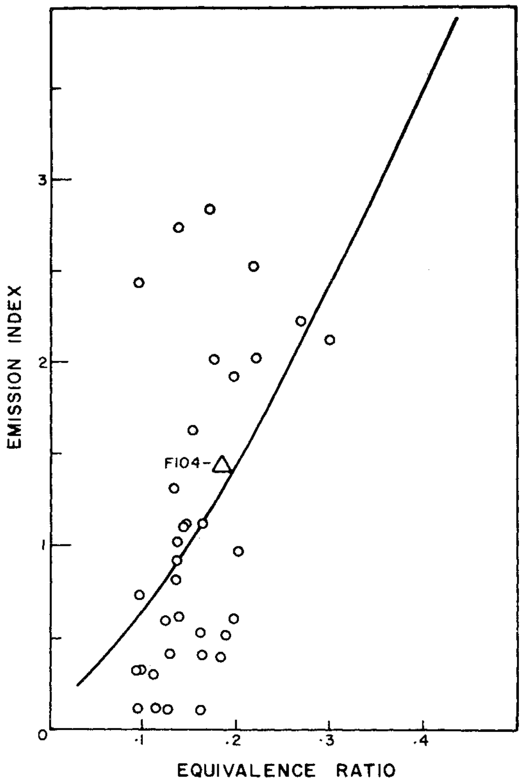

Fig. 4 The emission index obtained from this study (triangle) as compared with the results of a number of other studies conducted at ground level (circles). The smooth curve is a least-squares fit to the data.

lower limit for the piecewise continuous exponential distribution. The mass emission rates were calculated with different but reasonable size distributions to illustrate the sensitivity of the reported values to sizes outside the measurement range.

From Table 1, the emission index (lb aerosol per 1000 lb fuel) was calculated using $m = 4.00$ lb/hr and the fuel consumption rates supplied by the engine manufacturer. Figure 4 is a comparison of the emission index obtained from this study with that obtained from typical jet engines operating at ground level, as reported by Sawyer.[5] Table 1 implies that the uncertainty in the calculated value is probably about a factor of two.

In reporting exhaust emission levels, giving just the emission index provides an incomplete picture. The operating conditions of the engine must also be taken into account. Thus the emission index is more meaningful when it is given as a function of equivalence ratio (the actual fuel-to-air combustion ratio divided by the calculated stoichiometric fuel-to-air ratio) which is roughly proportional to the engine speed. For the cruise conditions reported here, the F104 has a fuel-to-air ratio of about 0.013. The stoichiometric fuel-to-air ratio for JP-4 fuel is about 0.068. The corresponding equivalence ratio is therefore 0.013/0.068, or 0.19.

Conclusions

The results of this study taken at face value indicate that the emission index of typical jet engines calculated from ground level measurements is comparable to the actual in-flight emission index for altitudes up to 30,000 ft. It appears that a properly instrumented aircraft can be used to obtain some worthwhile information concerning emissions at altitude but the size distribution should be more thoroughly investigated, both theoretically and experimentally.

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Slender Delta Wing with Conical Camber

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THE slenderness assumption of Munk[1] and Jones,[2] made over and above that of linearization, has often been used to get closed form analytical solutions for wings, bodies, and their combinations. In this Note, simple expressions are obtained for the aerodynamic characteristics of a slender delta wing, exhibiting an even-order polynomial twist distribution in the spanwise coordinate. In Ref. 3 is derived an integral equation which relates the twist distribution to the pressure distribution. For the slender wing it is

$$\Delta C_p = C_p^- - C_p^+$$
$$= \frac{4\,C_o}{\tan\delta\,(1-\eta^2)^{1/2}} + \frac{4\tan\delta}{\pi}\int_0^1 \frac{d\omega}{d\eta_i}\,\eta_i$$
$$\times \log\frac{(1-\eta^2)^{1/2}+(1-\eta_i^2)^{1/2}}{(1-\eta^2)^{1/2}-(1-\eta_i^2)^{1/2}}\,d\eta_i \qquad (1)$$

and

$$C_0 = \tan^2\delta\left[-\omega(1)+\frac{2}{\pi}\int_0^1\frac{d\omega}{d\eta}\sin^{-1}\eta\,d\eta\right] \qquad (2)$$

The twist distribution under consideration is of the type

$$\alpha_n(\eta) = -\omega_n(\eta) = -\epsilon_{0n} - \epsilon_n\,\eta^{2n}/2n \qquad (3)$$

Here C_p is the pressure coefficient, η the conical coordinate $y/s(x)$, α the twist distribution, $s(x)$ the local wing semispan, δ the wing semiapex angle, and + and − refer to the top and bottom surfaces of the wing, respectivey. The coordinate system is shown in Fig. 1.

Linearized theory predicts two kinds of solutions. One in which a leading edge singularity appears and the other in which the leading edge experiences zero load. Because

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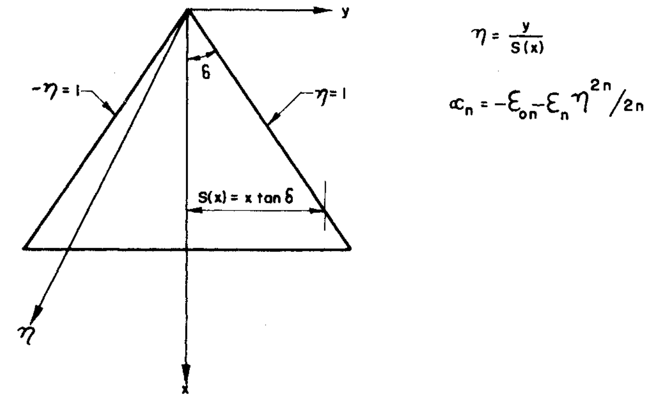

Fig. 1 Coordinate system.

the flow has a tendency to separate with the former type of flow, the latter type of solutions have been considered by several authors[4-6] to design wings of low drag. We also restrict the present analysis to this case and accordingly put $C_o = 0$. This will, therefore, relate the constants ϵ_{on} and ϵ_n.

On substituting Eq. (3) into Eq. (1) and integrating by parts, and writing down the integrals in terms of the Glauert integrals, we obtain

$$\Delta C_{pn} = \frac{4 \tan \delta}{2n+1} \epsilon_n (1-\eta^2)^{1/2} \sum_{i=0}^{n} J_i \eta^{2(n-i)} \tag{4}$$

where

$$\left.\begin{aligned} J_0 &= 1 \\ J_i &= \frac{1 \cdot 3 \cdots (2i-1)}{2 \cdot 4 \cdots (2i)} \end{aligned}\right\} \tag{5}$$

and from Eq. (2)

$$\epsilon_{0n} = -\frac{\epsilon_n}{2\pi n} \quad B\left(\frac{2n+1}{2}, \frac{1}{2}\right) \tag{6}$$

where B is the complete Beta function and is expressible in terms of the Gamma functions.

The spanwise distribution of lift is

$$L_n(\eta) = \int_{x_1}^{1} \Delta C_{p_n}(\eta_i)\, dx = \eta \int_{\eta}^{1} \Delta C_{p_n} \frac{d\eta_i}{\eta_i^2}$$

$$= \frac{2 \tan \delta}{n(2n+1)} \quad (1-\eta^2)^{3/2} \sum_{m=0}^{n-1} (2m+1) J_m \eta^{2(n-m-1)} \tag{7}$$

Notice here that unlike the plane delta which shows infinite slope for $L(\eta)$ at the wing tip, Eq. (7) shows zero slope.

The lift and drag coefficients also reduce to simple expressions. These are

$$C_{L_n} = -\int_0^1 \Delta C_{p_n}\, d\eta$$

$$= -\frac{\pi \tan \delta}{(2n+1)} \epsilon_n \sum_{k=0}^{n} \frac{J_k J_{n-k}}{(n-k+1)} \tag{8}$$

and

$$C_{D_{n,m}} = -\int_0^1 \Delta C_{p_n} \omega_m\, d\eta$$

$$= -\epsilon_n \epsilon_m \frac{\pi \tan \delta}{(2n+1)} \sum_{k=0}^{n} \frac{J_k J_{n+m-k}}{(n+m-k+1)} \tag{9}$$

$$- \epsilon_{0m} C_{L_n}$$

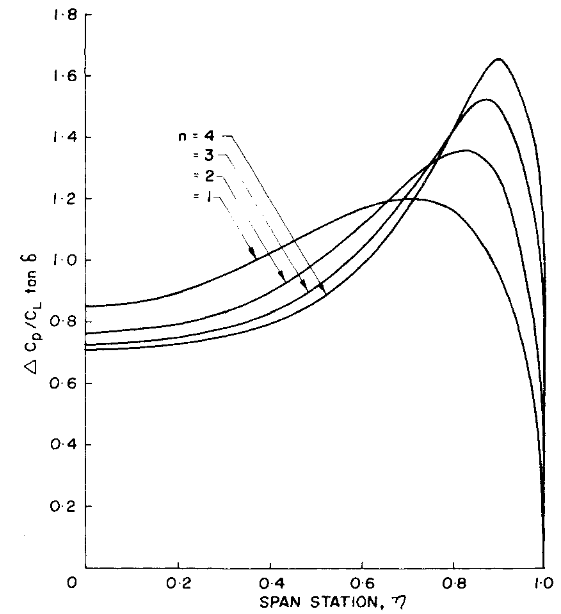

Fig. 2a Pressure distribution on slender delta with attachment line at the leading edge.

For a delta wing the chordwise distribution of lift will be linear and the centre of pressure is fixed at ⅔ root chord from the wing apex. The pitching moment coefficient about the wing apex is

$$C_{M_n} = -2/3\ C_{L_n} \tag{10}$$

The wing ordinate distribution is given by

$$\frac{\partial z}{\partial \eta} = \bar{z} - \eta \frac{\partial \bar{z}}{\partial \eta} = \omega(\eta) \tag{11}$$

where

$$z = x\, \bar{z}(\eta)$$

Integrating (11) gives

$$\bar{z}(\eta) = K\eta - \eta \int \frac{\omega(\eta)}{\eta^2}\, d\eta \tag{12}$$

where K is an arbitrary constant and shows that the wing shapes are nonunique to the extent that a dihedral angle does not affect the aerodynamic characteristics in the linearized theory. Without any loss of generality, we put $K = 0$, and the substitution of Eq. (3) into Eq. (12) then yields

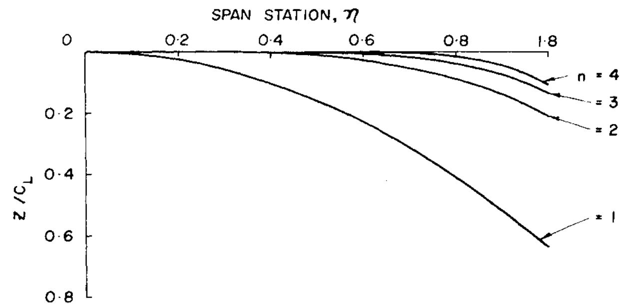

Fig. 2b Ordinate distribution of slender delta.

$$z_n = -\frac{x\,\epsilon_n\,\eta^{2n}}{2n\,(2n-1)} - \epsilon_{0n}\,x \tag{13}$$

These results may be extended to nondelta planforms in the same way as described by Jones.[2] However, it is always useful to remember the drawbacks of any analysis when several idealizations have been made to arrive at simple solutions. Thus the slenderness assumption violates the Kutta condition in subsonic flow by ignoring the presence of the trailing edge. As a consequence, the lift near the trailing edge is considerably overestimated. An emperical correction suggested by Laidlaw and Hsu[7] when used with the slender wing results was found to be very useful and is therefore repeated here

$$\Delta C_{p_c}\,(x,\ \eta) = \sqrt{1 - \left(\frac{x}{c}\right)^2}\ \Delta C_{p_i} \tag{14}$$

where ΔC_{pc} is the corrected value, ΔC_{pi} is from slender wing theory, and c is the local chord. The result is valid for steady and unsteady cases and has been successfully used in flutter analysis. In supersonic flow, the presence of the trailing edge is felt only through the boundary layer and for most purposes it is sufficient to assume that pressure equalization takes place through an oblique shock. The slender wing solution is therefore valid quite close to the trailing edge. The slender theory, however, fails to predict the wave drag.

As a matter of interest, the spanwise pressure and ordinate distributions are shown in Fig. 2 for n = 1–4. It is seen that these wings have a leading edge droop which helps the flow to turn smoothly, and hence, the predicted aerodynamic properties may be expected to be close to reality.

The analytical results of this note augment the solutions of Ref. 2 for the flat delta, and of Ref. 3 for the delta with a linear twist distribution. The simplicity of the results along with the Laidlaw-Hsu correction should make them useful where quick estimates of the pressure distribution are required.

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Influence of Static Aeroelasticity on Oblique Winged Aircraft

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Introduction

BENDING deformation has an important influence on the static aeroelastic behavior of slender wings of moderate sweep. Bending deformation is responsible for the strong tendency of sweptforward wings to diverge and accounts for the fact that moderately sweptback wings will not diverge. Spanwise aerodynamic load redistribution is caused, in part, by bending deformation and accounts for the poor aileron reversal characteristics of sweptback wings. Interest has recently been expressed about the impact of static aeroelasticity on the lateral control of oblique winged aircraft.[1,2] The purpose of this Note is to illustrate, by use of a simple example, together with results from published literature, an aeroelastic phenomenon which the author believes to be unique to oblique winged aircraft. This phenomenon is the aeroelastic roll moment and occurs because upward bending deflection of the sweptforward wing generates additional lift while the converse is true of sweptback wings.

Discussion

For problems involving swept wing static aeroelasticity, bending and torsional deformations are coupled. Excellent discussions of the formulation of such problems may be found, for instance, in Refs. 3–5. The essential features of the interaction between wing deformation, aerodynamic forces and aileron trimming can be illustrated by means of a simple example. Consider the constant chord, slender, oblique wing shown in Fig. 1. This elastically symmetrical wing model is swept at an angle Λ to the airstream and has full-span ailerons for lateral control. Pitch, yaw, and plunge degrees of freedom are not permitted, but, as will be shown, the ailerons must be deflected to guarantee static equilbrium in roll.

To simplify the aeroelastic analysis, assume that the wing is uncambered and that the straight elastic axis and the line of aerodynamic centers coincide. In this case, only

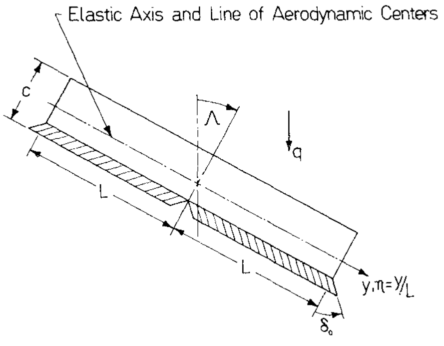

Fig. 1. Oblique wing idealization.

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